Chapter 7 Straight-line Graph

Part 2

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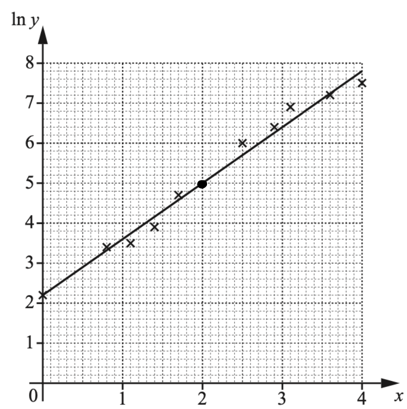
1. The relationship between experimental values of two variables, *x* and *y*, is given by $y = Ab^x$, where *A* and *b* are constants.
   a. Transform the relationship $y = Ab^x$ into straight line form.

$\ln y = \ln A b^x$ [2]

$\ln y = \ln A + x \ln b$

The diagram shows ln *y* plotted against *x* for ten different pairs of values of *x* and *y*. The line of best fit has been drawn.



(2,5) (0,2.2)

   b. Find the equation of the line of best fit and the value, correct to 1 significant figure, of *A* and of *b.*

$m = \frac{2.2-5}{0-2} = 1.4$ $\quad \ln b = 1.4$ [4]

$b = e^{1.4}$

$\ln y = 1.4x + \ln A$ $\quad = 4$

$2.2 = \ln A$ $\quad A = 9$

$e^{2.2} = A$

   c. Find the value, correct to 1 significant figure, of *y* when *x* = 2.7.

$y = Ab^x$ [2]

$= 9 \times 4^{2.7} = 380 \approx 400$ (1 s.f)

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2. When lg y is plotted against x^2 a straight line graph is obtained which passes through the points (2, 4) and (6, 16).

x^2 — lg y

a. Show that $y = 10^{A+Bx^2}$, where A and B are constants. [4]

$m = \frac{16-4}{6-2} = 3$

$\lg y = 3x^2 + c$

$4 = 3 \times 2 + c$

$4 = 6 + c$

$c = -2$

$\lg y = 3x^2 - 2$

$y = 10^{3x^2-2}$

b. Find y when $x = \frac{1}{\sqrt{3}}$. [2]

$y = 10^{3 \times \frac{1}{3} - 2} = 10^{-1} = \frac{1}{10}$

c. Find the positive value of x when $y = 2$. [3]

$y = 10^{3x^2-2}$

$2 = 10^{3x^2-2}$

$\lg 2 = \lg 10^{3x^2-2}$

$3x^2 - 2 = \lg 2$

$x^2 = 0.767$

$x = 0.876$

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3. When e^y is plotted against $\frac{1}{x}$, a straight line graph passing through the points (2, 20) and (4, 8) is obtained.

   a. Find y in terms of x. [5]

$$m = \frac{20-8}{2-4} = \frac{12}{-2} = -6$$

$$e^y = -6\frac{1}{x} + c$$

$$8 = -6 \times 4 + c$$

$$8 = -24 + c$$

$$c = 32$$

$$e^y = \frac{-6}{x} + 32$$

$$y = \ln\left(-\frac{6}{x} + 32\right)$$

   b. Hence find the positive values of x for which y is defined. [1]

$$-\frac{6}{x} + 32 > 0 \quad -\frac{6}{x} > -32 \quad \frac{6}{x} < 32 \quad \frac{6}{32} < x \quad x > \frac{3}{16}$$

   c. Find the exact value of y when $x = 3$. [1]

$$y = \ln(-2 + 32)$$

$$= \ln 30$$

   d. Find the exact value of x when $y = 2$. [2]

$$2 = \ln\left(-\frac{6}{x} + 32\right)$$

$$e^2 - 32 = -\frac{6}{x}$$

$$\frac{6}{x} = 32 - e^2$$

$$x = \frac{6}{32 - e^2}$$

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4. When lg y is plotted against x, a straight line graph passing through the points (2.2,3.6) and (3.4,6) is obtained.
   a. Given that $y = Ab^x$, find the value of each of the constants A and b.

[5]

$$m = \frac{3.6 - 6}{2.2 - 3.4}$$

$$= 2$$

$$\lg y = 2x + c$$

$$6 = 2 \times 3.4 + c$$

$$6 = 6.8 + c$$

$$c = -0.8$$

$$\lg y = 2x - 0.8$$

$$y = 10^{2x - 0.8}$$

$$= 10^{2x} \times 10^{-0.8}$$

   b. Find x when y = 900.

[2]

$$900 = 10^{2x - 0.8}$$

$$\lg 900 = 2x - 0.8$$

$$2x = \lg(900) + 0.8$$

$$x = 1.88$$

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5. When $log\ y^2$ is plotted against *x*, a straight line is obtained passing through the points (5, 12) and (3, 20). Find *y* in terms of *x*, giving your answer in the form $y = 10^{ax+b}$ where *a* and *b* are integers.

[5]

$$m = \frac{20-12}{3-5} = -4$$

$$\lg y^2 = -4x + c$$

$$20 = -12 + c$$

$$c = 32$$

$$\lg y^2 = -4x + 32$$

$$\lg y = -2x + 16$$

$$y = 10^{-2x+16}$$

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6.

x	1	1.5	2	2.5	3
y	6	14.3	48	228	1536
x^2	1	2.25	4	6.25	9
$\lg y$	0.78	1.16	1.68	2.36	3.19

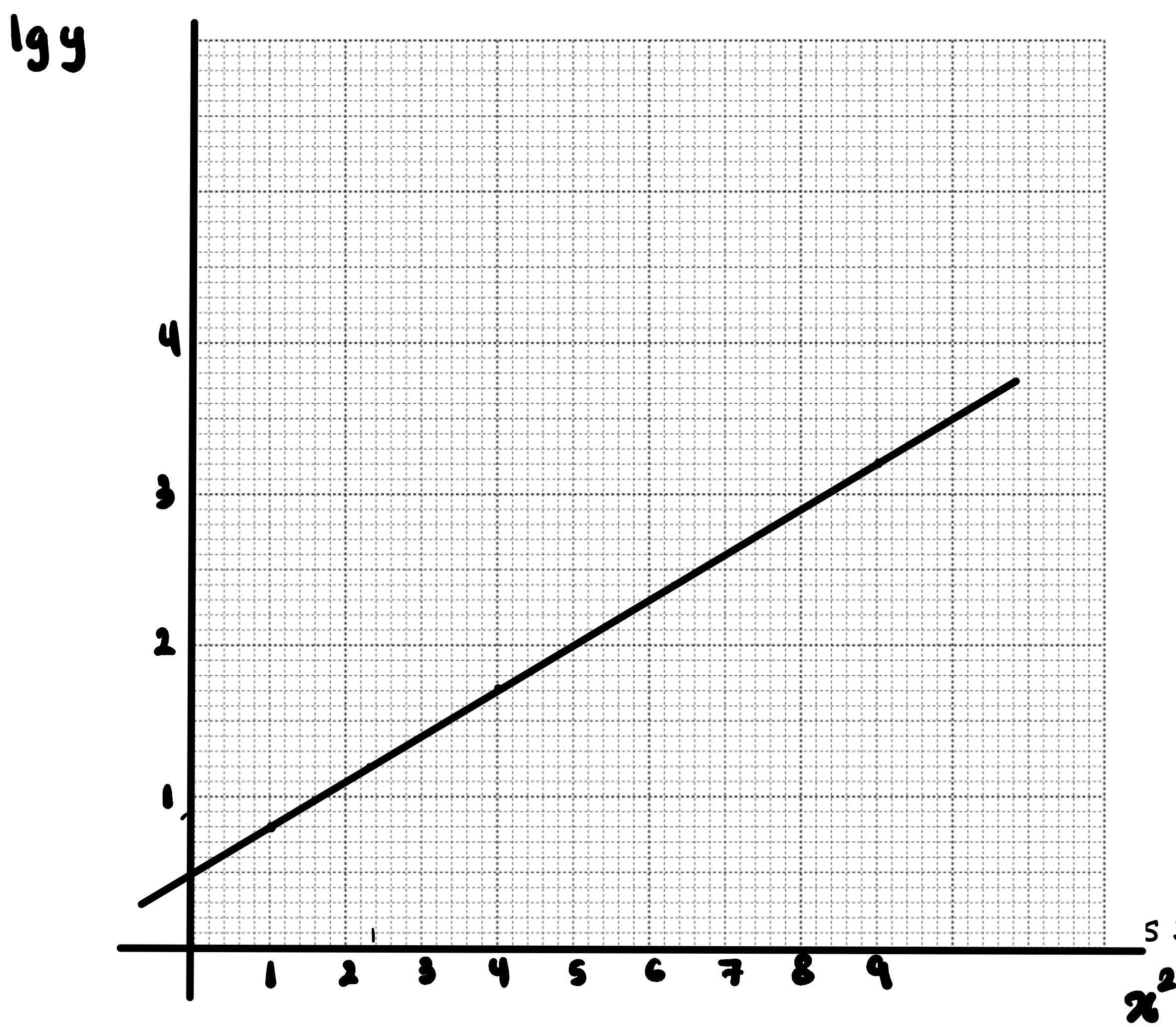
$y = Ab^{x^2}$

$\lg y = \lg A + x^2 \lg b$

The table shows values of the variables *x* and *y* such that $y = Ab^{x^2}$, where *A* and *b* are constants.

(i) Draw a straight line graph to show that $y = Ab^{x^2}$.

[4]



(ii) Use your graph to find the value of A and of b.

[4]

$\lg A = 0.5$

$A = 10^{0.5}$

$= 3.16$

$m = \dfrac{1.16 - 0.78}{2.25 - 1}$

$= 0.304$

$\lg b = 0.304$

$b = 10^{0.304}$

$= 2.01$

(iii) Estimate the value of x when $y = 100$.

[2]

$y = Ab^{x^2}$

$100 = 3.16 \times 2.01^{x^2}$

$31.65 = 2.01^{x^2}$

$\lg 31.65 = \lg 2.01^{x^2}$

$\lg 31.65 = x^2 \lg 2.01$

$x = 2.22$